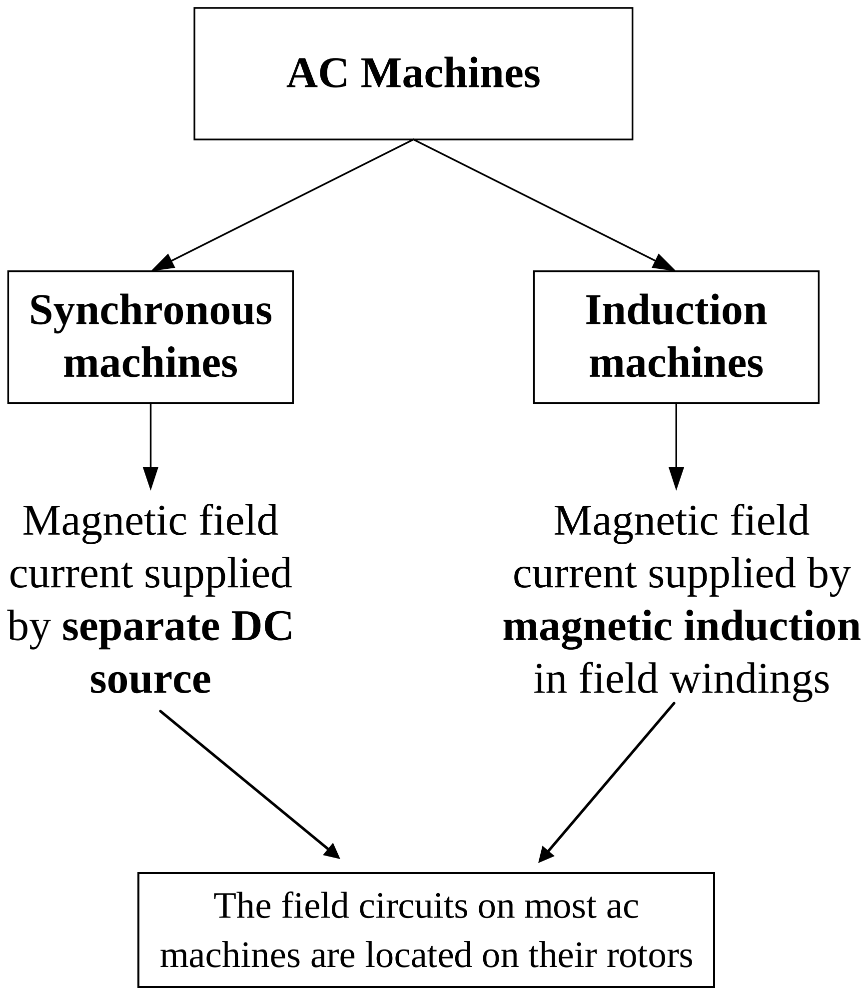
Chapter 4.0 AC Machinery Fundamentals

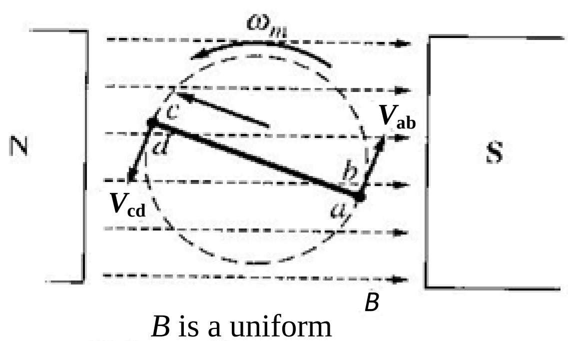
AC machines are made up of:

1. **Generators** – converts mechanical energy to electrical energy
2. **Motors** – converts electrical energy to mechanical energy



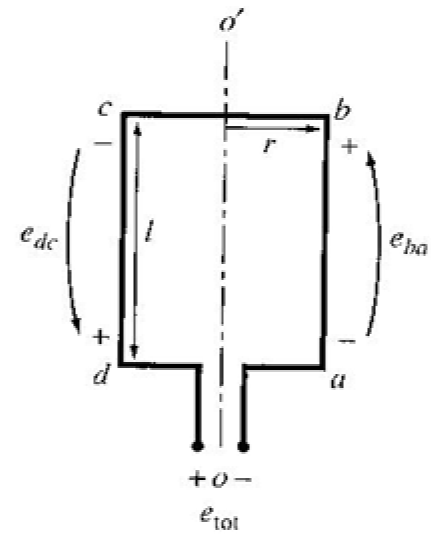
4.1 A Simple Loop in a Uniform Magnetic Field

The figure below shows a simple rotating loop in a uniform magnetic field.



B is a uniform magnetic field aligned as shown

Front View



View of Coil

Stator = stationary part of machine
Rotor = rotating part of machine

4.1.1 The Voltage Induced in a Simple Rotating Loop

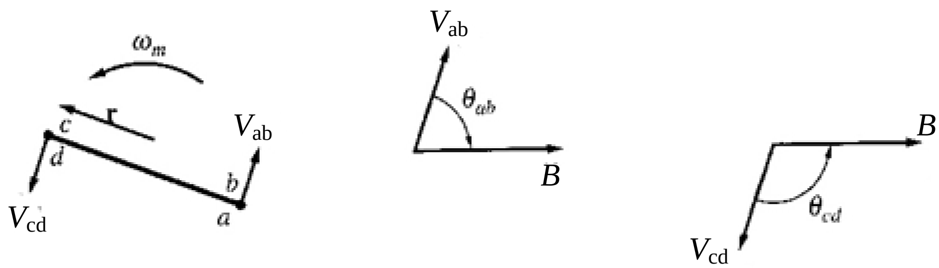
Rotor rotation will **induce a voltage** in the wire loop. To determine **total voltage induced** in loop e_{tot} need to examine each segment **separately**.

The induced voltage on each segment is calculated as:

$$e_{\text{ind}} = (\bar{v} \times \bar{B}) \cdot \bar{l},$$

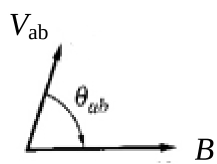
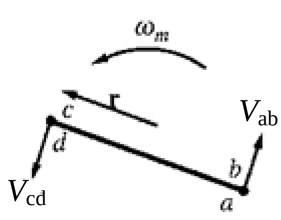
where $\bar{v}$ is the velocity of the wire that is tangential to the path of rotation
$\bar{B}$ is the magnetic flux density vector
$\bar{l}$ length of each segment which plane of action depends on the side

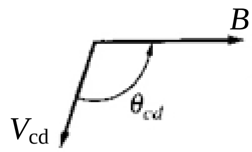
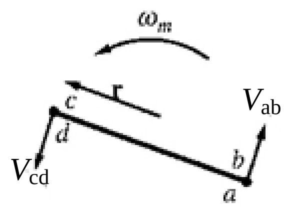
If the velocity and orientation of the sides of the loop w.r.t. the magnetic field is as shown in (a), then the direction of motion w.r.t. to the magnetic field is shown in (b) and (c) for sides *ab* and *cd*, respectively.



(a) (b) (c)

The analysis of induced voltage in each segment can be approached as follows:

Segment *ab* 	• The velocity of the of the wire is tangential to the path of rotation • B points to the right • $\bar{v} \times \bar{B}$ points **into the page** which is the same direction as segment *ab* ∴ Induced voltage on this segment is $e_{\text{ba}} = (\bar{v} \times \bar{B}) \cdot \bar{l}$ $= V_{\text{ab}} Bl\, sin\theta_{\text{ab}}$ into the page
Segment *bc* 	• The first half of this segment $\bar{v} \times \bar{B}$ points **into the page** • The second half of this segment $\bar{v} \times \bar{B}$ points **out of the page** • **Length** l is in the plane of the page • Dot product of l with $\bar{v} \times \bar{B}$ is zero since the planes are perpendicular ∴ The induced voltage on this segment is $e_{\text{cb}} = 0$

Segment *cd* 	• The velocity of the of the wire is tangential to the path of rotation • B points to the right • $\bar{v} \times \bar{B}$ points ___________________ ∴The induced voltage on this segment is $e_{\mathrm{ba}} = (\bar{v} \times \bar{B}) \cdot \bar{l}$
Segment *da* 	• Just as in segment *bc*, the dot product of *l* with $\bar{v} \times \bar{B}$ is zero since the planes are perpendicular ∴ The induced voltage on this segment is ______________________

The **total induced voltage** in the loop is,

$$\text{Total } e_{\mathrm{ind}} = e_{\mathrm{ba}} + e_{\mathrm{cb}} + e_{\mathrm{dc}} + e_{\mathrm{ad}}$$

$$= vBl \sin\theta_{\mathrm{ba}} + vBl \sin\theta_{\mathrm{dc}}$$

$$= 2vBl \sin\theta$$

Note that $\sin\theta_{\mathrm{ba}} = \sin(180 - \theta_{\mathrm{dc}})$ and the trigonometric function identity $\sin\theta = \sin(180 - \theta)$.

We know that if the loop is rotating with a constant angular velocity *ω*, then

$$\theta = \omega t$$

And the tangential velocity of the edges of the loop is

$$v = r\omega,$$

where *r* is the radius from axis of rotation out to the edge of the loop and *ω* is the angular velocity of the loop.

Therefore, the induced voltage can be alternatively expressed as

$$\begin{aligned} e_{\text{ind}} &= 2rBl\omega \sin \omega t \\ &= A_{\text{loop}} B\omega \sin \omega t \\ &= \phi_{\max} \omega \sin \omega t \end{aligned}$$

Thus, *the voltage generated in the loop is a sinusoid whose magnitude is equal to the product of the flux inside the machine and the speed of rotation of the machine*.

The voltage in any real ac machines will depend on three factors:

- Flux level (***B*** **or** $\boldsymbol{\phi}$)
- Speed of Rotation (**v or** $\boldsymbol{\omega}$)
- Machine Constants (**length *l* and machine materials**)

4.1.2 The Torque Induced in a Current-Carrying Loop

Assuming that the loop is at an arbitrary angle θ to the magnetic field and a **current *i* is flowing** in it. Then a torque will be induced in the wire loop.

Once more, analysis on each segment of the loop needs to be carried out.

The force on each segment of the loop is given as (Chapter 1)

$$F = i(\bar{l} \times \bar{B})$$

where *i* is the magnitude of current in the segment

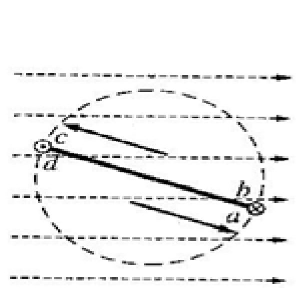
$\bar{l}$ is the length of the segment, with the direction of *l* defined to be in the direction of the current flow

$\bar{B}$ is the magnetic flux density vector

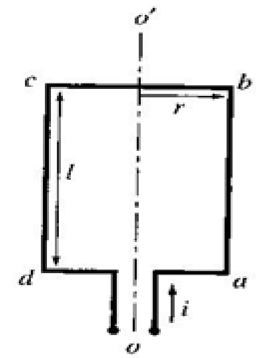
The torque on that segment will then be given as

$$\begin{aligned} \tau &= \text{(force applied)(perpendicular distance from the axis of rotation)} \\ &= (F)(r \sin \theta) \\ &= rF \sin \theta \end{aligned}$$

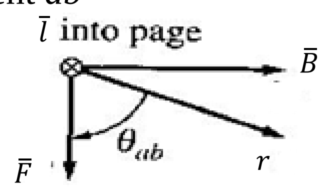
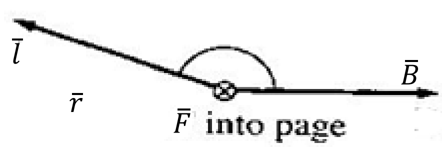
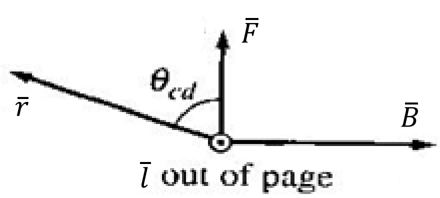
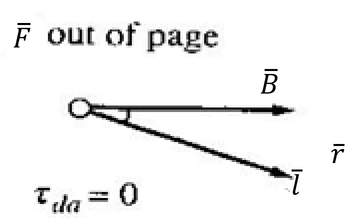
where θ is the angle between the vector *r* and the vector *F*.



(a)



(b)

Segment *ab* 	• Direction of current is into the page • The induced force is $F = i(\bar{l} \times \bar{B}) = ilB$ **down** • The resulting torque is $\tau_{\mathrm{ab}} = rF \sin\theta_{\mathrm{ab}}$ $= rilB \sin\theta_{\mathrm{ab}}$ **clockwise**
Segment *bc* 	• The direction of current is in the plane of the page • The induced force $F = i(\bar{l} \times \bar{B}) = ilB$ **into the page** • $\bar{r}$ and $\bar{l}$ are in parallel • The resulting torque $\tau_{bc} = 0$
Segment *cd* 	• Direction of current is ______________ • The induced force is $F = i(\bar{l} \times \bar{B}) = ilB$ __________ • $\tau_{\mathrm{cd}} = rilB \sin\theta_{\mathrm{cd}}$ **clockwise**
Segment *da* 	• The direction of current is in the plane of the page • $\bar{r}$ and $\bar{l}$ are in parallel • $\theta_{\mathrm{da}} = 0$

The total induced torque on the loop, τ_{ind} is the sum of the torques on each of its sides:

$$\tau_{\text{ind}} = \tau_{\text{ab}} + \tau_{\text{bc}} + \tau_{\text{cd}} + \tau_{\text{da}}$$

$$= 2rilB\sin\theta$$

Note that $\theta_{\text{ab}} = \theta_{\text{cd}}$.

To relate behaviour of the single loop to the behaviour of larger ac machines, alternative expression for induced torque is derived.

The current flowing in the wire loop will generate a magnetic flux density $\bar{B}_{\text{loop}}$:

$$\bar{B}_{\text{loop}} = \frac{\mu i}{G}$$

where G = factor depending on loop geometry.

Since loop area $A = 2rl$,

$$\tau_{\text{ind}} = 2rilB\sin\theta$$
$$= A(\frac{G}{\mu}B_{\text{loop}})B_{\text{s}}\sin\theta$$
$$\therefore\ \tau_{\text{ind}} = kB_{\text{loop}}B_{\text{S}}\sin\theta$$

Or

$$\tau_{\text{ind}} = k(\bar{B}_{\text{loop}} \times \bar{B}_{\text{S}})$$

where $k = {}^{AG}/_{\mu}$,

$\bar{B}_{\text{loop}}$, $\bar{B}_{\text{s}}$ = the magnetic field generated by the rotor, the stator magnetic field
θ = angle between the rotor and stator magnetic fields

Generally, the **torque** in real machines **depends** on:

- strength of the **rotor magnetic field**
- strength of **external (stator) magnetic field**
- **angle** between the two fields
- **Machine constants**

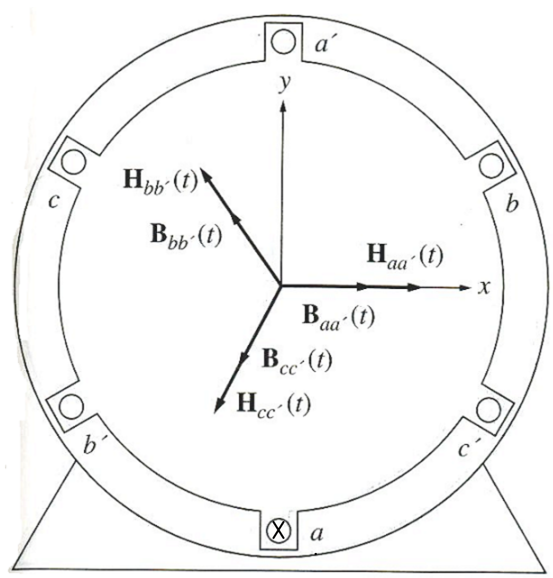
4.2 The Rotating Magnetic Field

If one magnetic field is produced by the stator of an ac machine and the other by the rotor, then **a torque** will be **induced** in the rotor which will **cause the rotor to turn** and **align the rotor magnetic field with the stator magnetic field**.

If there were some way to make the stator magnetic field rotate, then the induced torque in the rotor would cause it to 'chase' the stator magnetic field.

How to create a rotating stator magnetic field?

Use a set of **three-phase windings displaced by 120° electrical** around the machine circumference.



The current in coil *aa'* flows into the *a* end of the coil and flow out of the *a'* end of the coil. The three-phase currents are:

$$i_{aa'}(t) = I_M \sin \omega t \text{ A}$$
$$i_{bb'}(t) = I_M \sin(\omega t - 120°) \text{ A}$$
$$i_{cc'}(t) = I_M \sin(\omega t - 240°) \text{ A}$$

Each coil will produce magnetic field intensity:

$$\overline{H}_{aa'}(t) = H_M \sin \omega t \angle 0° \text{ A} \cdot \text{turns/m}$$
$$\overline{H}_{bb'}(t) = H_M \sin(\omega t - 120°) \angle 120° \text{ A} \cdot \text{turns/m}$$
$$\overline{H}_{cc'}(t) = H_M \sin(\omega t - 240°) \angle 240° \text{ A} \cdot \text{turns/m}$$

Notice that the **magnitude varies in sinusoidal** with time but the **direction** is always **constant**.

The **resulting flux densitites** are then given by,

$$\bar{B}_{aa'}(t) = B_{\mathrm{M}} \sin \omega t \angle 0^\circ \text{ T}$$
$$\bar{B}_{bb'}(t) = B_{\mathrm{M}} \sin(\omega t - 120^\circ)\angle 120^\circ \text{ T}$$
$$\bar{B}_{cc'}(t) = B_{\mathrm{M}} \sin(\omega t - 240^\circ)\angle 240^\circ \text{ T}$$

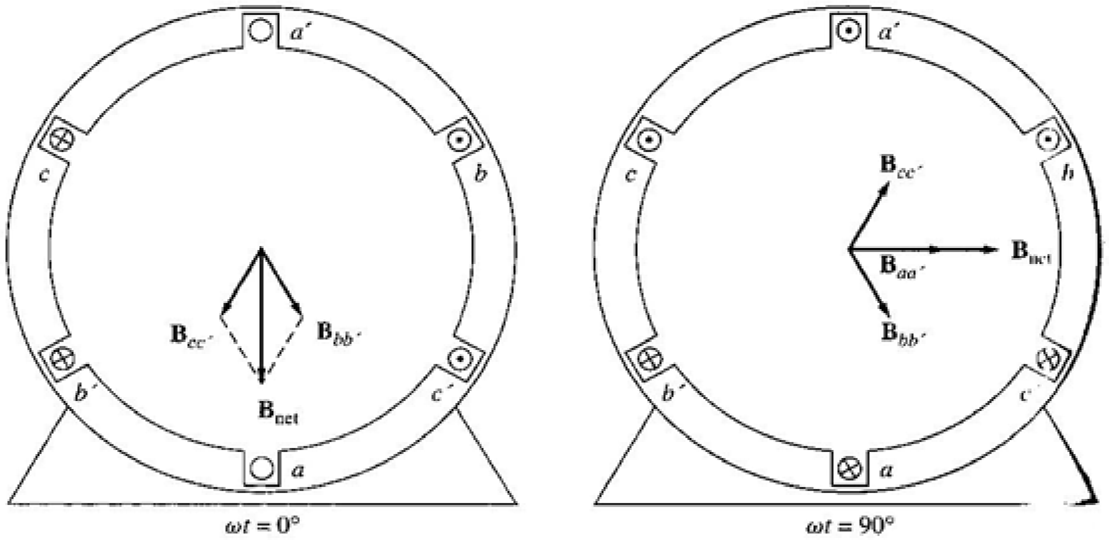
where $B_{\mathrm{M}} = \mu H_{\mathrm{M}}$

In order to observe the net field rotation, let us examine the flux densities at specific times:

$\omega t = 0^\circ$	$\omega t = 90^\circ$
$\bar{B}_{aa'}(t) = 0$ $\bar{B}_{bb'}(t) = B_{\mathrm{M}} \sin(-120^\circ)\angle 120^\circ$ T $\bar{B}_{cc'}(t) = B_{\mathrm{M}} \sin(-240^\circ)\angle 240^\circ$ T The total magnetic field from all the three coils is $\bar{B}_{\mathrm{net}} = \bar{B}_{aa'} + \bar{B}_{bb'} + \bar{B}_{cc'}$ $= 1.5B_{\mathrm{M}}\angle -90^\circ$	$\bar{B}_{aa'}(t) = B_{\mathrm{M}}\angle 0^\circ$ T $\bar{B}_{bb'}(t) = B_{\mathrm{M}} \sin(-30^\circ)\angle 120^\circ$ T $\bar{B}_{cc'}(t) = B_{\mathrm{M}} \sin(-150^\circ)\angle 240^\circ$ T The total magnetic field from all the three coils is $\bar{B}_{\mathrm{net}} = \bar{B}_{aa'} + \bar{B}_{bb'} + \bar{B}_{cc'}$ $= 1.5B_{\mathrm{M}}\angle 0^\circ$

Field magnitude constant, direction changes! Rotating magnetic field.

Therefore the net magnetic field is the effect of the sum of the magnetic fields produced by the three-phase currents in the stator.



4.2.1 Proof of the Rotating Magnetic Field Concept

As seen previously, at anytime *t* the magnitude of the magnetic flux density is the same and it is $1.5B_M$. It will continue to rotate at angular velocity ω.

The **total magnetic flux density** in the stator is obtained from **vectorial addition** of the three magnetic field components, i.e.

$$\bar{B}_{\text{net}}(t) = \bar{B}_{aa'}(t) + \bar{B}_{bb'}(t) + \bar{B}_{cc'}(t)$$
$$= B_M \sin \omega t \angle 0° + B_M \sin(\omega t - 120°)\angle 120° + B_M \sin(\omega t - 240°)\angle 240°$$

Each of the three component magnetic fields can now be broken down into its *x* and *y* components.

Note that, here we consider the *x* direction is to the right and the *y* direction is upward. Therefore,

$$\bar{B}_{\text{net}}(t) = B_M \sin \omega t\, \hat{x}$$
$$-[0.5\bar{B}_M \sin(\omega t - 120°)]\hat{x} + \left[\frac{\sqrt{3}}{2}\bar{B}_M \sin(\omega t - 120°)\right]\hat{y}$$
$$-[0.5\bar{B}_M \sin(\omega t - 240°)]\hat{x} - \left[\frac{\sqrt{3}}{2}\bar{B}_M \sin(\omega t - 240°)\right]\hat{y}$$

By the angle-addition trigonometric function, the net magnetic flux density is

$$\bar{B}_{\text{net}}(t) = (1.5B_M \sin \omega t)\hat{x} - (1.5B_M \cos \omega t)\hat{y}$$

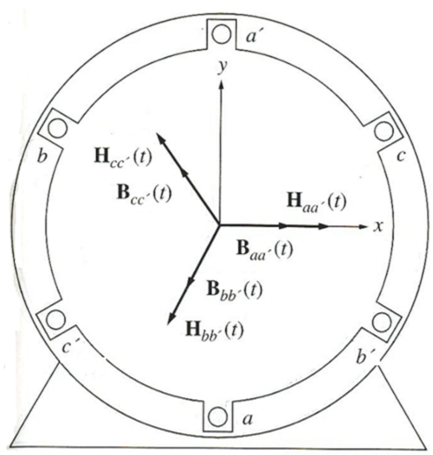
At $\omega t = 0°$, $\bar{B}_{\text{net}} = 1.5B_{\text{M}}\angle -90°$ and that at $\omega t = 90°$, $\bar{B}_{\text{net}} = 1.5B_{\text{M}}\angle 0°$

Field magnitude is constant
Field direction is counter-clockwise with at an angular velocity ω.

This proofs that a *three-phase set of currents*, *each of equal magnitude and differing in phase by* $120°$, *flows in a three-phase winding, it will produce a rotating magnetic field of constant magnitude*.

Reversing the Direction of the Magnetic Field Rotation

If the current in any two of the three coils is swapped, the direction of the magnetic field's rotation will be reversed. Switch phases bb' and cc' (i.e. current in $bb'$is delayed by $240°$ while that of cc' is delayed by $120°$ and calculate the resulting flux density. Note that physical winding positions of phases *b* and *c* do not change in the stator.



$$\bar{B}_{\text{net}} = B_{\text{M}} \sin \omega t \angle 0° + B_{\text{M}} \sin(\omega t - 240°)\angle 120° + B_{\text{M}} \sin(\omega t - 120°)\angle 240°$$

Expand the net magnetic flux density to its *x* and *y* components....

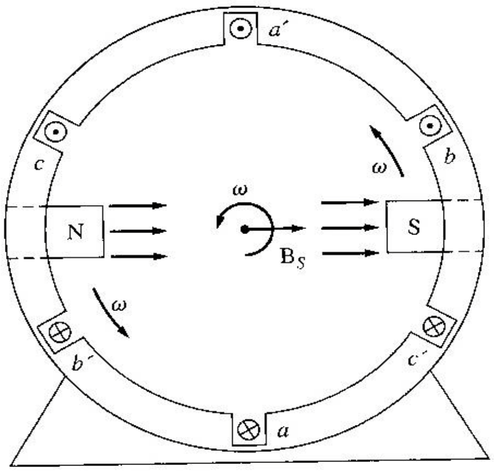
You should obtain.

$$\bar{B}_{\text{net}}(t) = (1.5B_{\text{M}} \sin \omega t)\hat{x} + (1.5B_{\text{M}} \cos \omega t)\hat{y}$$

Did you notice that the magnetic field has the **same magnitude** but rotates in a **clockwise direction**?

4.2.2 The Relationship between Electrical Frequency and the Speed of Magnetic Field Rotation

In the three phase coils considered, the stator is said to be **two-pole** and occurs in the following **counterclockwise** order:



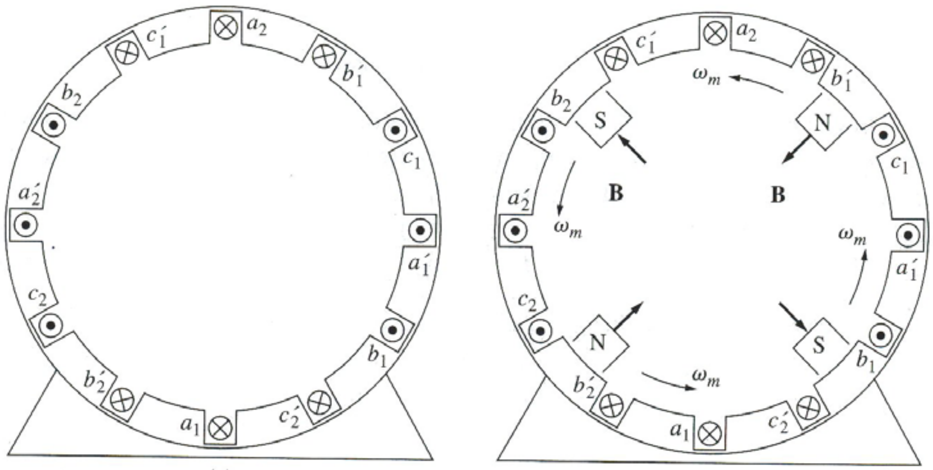
a – c' – b - a'– c – b'

Poles complete ________ mechanical rotation in ________ electrical cycle of current.

Hence, the **relation between the electrical and mechanical components** are given by:

$$\theta_e = \theta_m$$
$$f_e = f_m$$
$$\omega_e = \omega_m$$

Where θ_m, f_m, and ω_m are the mechanical angle, speed in revolutions per second, and speed in radians per second, respectively. While θ_e, f_e, and ω_e are the electrical angle, electrical speed in hertz, and electrical speed in radians per second respectively.



(a) A simple four-pole stator winding. (b) The resulting stator magnetic poles. Notice that there are moving poles of alternating polarity every 90° around the stator surface.

If we repeat the winding pattern twice, we have a four pole machine:
a – c' – b - a'– c – b'- a – c' – b - a'– c – b'

Poles complete **1/2** a mechanical rotation in **1** electrical cycle of current. Hence, the electrical and mechanical components related by:

$$\theta_\mathrm{e} = 2\theta_\mathrm{m}$$
$$f_\mathrm{e} = 2f_\mathrm{m}$$
$$\omega_\mathrm{e} = 2\omega_\mathrm{m}$$

Therefore, in general, for a *P* pole stator,

$$\theta_\mathrm{e} = {}^{P}\!/_{2}\,\theta_\mathrm{m}$$

$$f_\mathrm{e} = {}^{P}\!/_{2}\,f_\mathrm{m}$$

$$\omega_\mathrm{e} = {}^{P}\!/_{2}\,\omega_\mathrm{m}$$

Since $f_\mathrm{m} = \frac{n_\mathrm{m}}{60}$ and n_m is the mechanical speed of the magnetic field in revolutions per minute, thus the relationship of the electrical frequency in hertz to the resulting mechanical speed of the magnetic fields in revolutions per minute is

$$f_\mathrm{e} = \frac{n_\mathrm{m}P}{120}$$

4.3 Magnetomotive force and flux distribution in AC machines

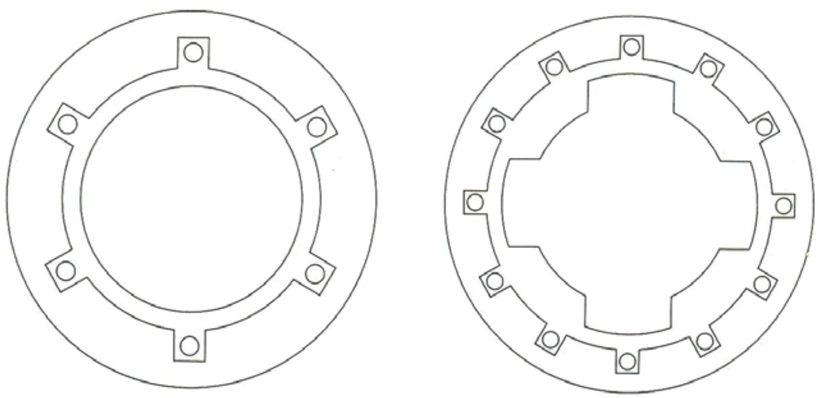
In Section 4.2, the following **assumptions** were made:

- flux produced inside ac machine is in free space
- direction of flux density is perpendicular to plane of coil (given by right hand rule)

Not true for real AC machines!
Because of **ferromagnetic rotor in centre** of machine with a small airgap between the stator and the rotor.

The **rotor** can be:

(a) cylindrical (nonsalient-pole)
(b) salient-pole

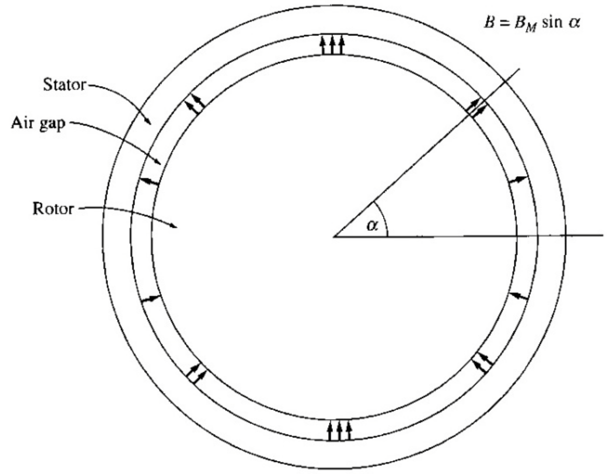


(a) (b)

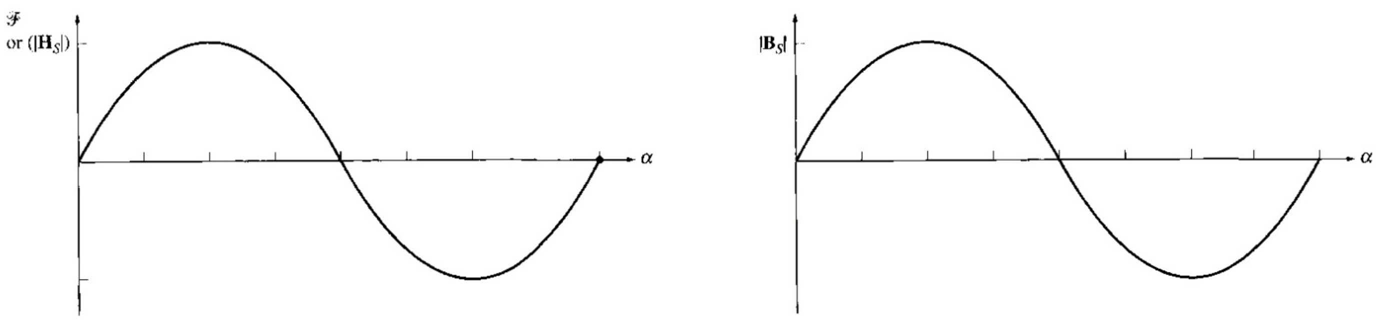
Only cylindrical rotors are considered in this course.
In the machine, $R_{\text{gap}} \gg R_{\text{stator}}, R_{\text{rotor}}$

Therefore, **flux density** vector $\bar{B}$ takes **shortest possible path** across the air gap i.e. perpendicular between rotor and stator.

To produce **sinusoidal voltage** in machine, **requires the magnitude of vector $\bar{\boldsymbol{B}}$** to vary in a **sinusoidal manner.** This occurs **only when magnetising intensity** $\bar{H}$(and mmf, F) is **sinusoidal along air gap surface.**



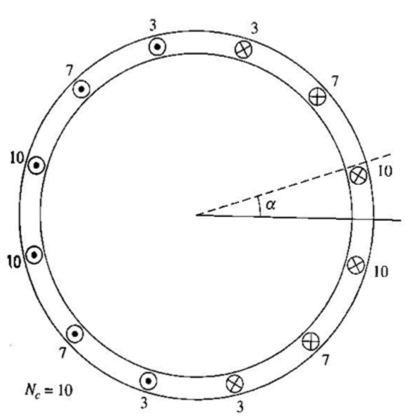
Cylindrical rotor with sinusoidally varying air gap flux density



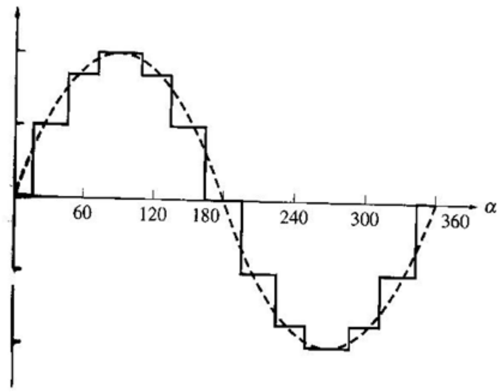
MMF or $|\bar{H}_S|$and $|\bar{B}_S|$as a function of angle in the air

The most straight forward way to achieve a sinusoidal variation of magnetomotive force along the surface of the air gap is to distribute the turns of the winding.

For example:



Distributed stator winding designed to produce a sinusoidally varying air gap flux density



The mmf distribution resulting from the winding, compared to an ideal distribution.

The number of conductors in each slot is:

$$n_{\mathrm{c}} = N_{\mathrm{C}} \cos \alpha$$

where N_C = number of conductors at an angle 0°.

Increasing number of slots and making them **closely spaced** gives a **better approximation** of sinusoidal distribution of mmf.

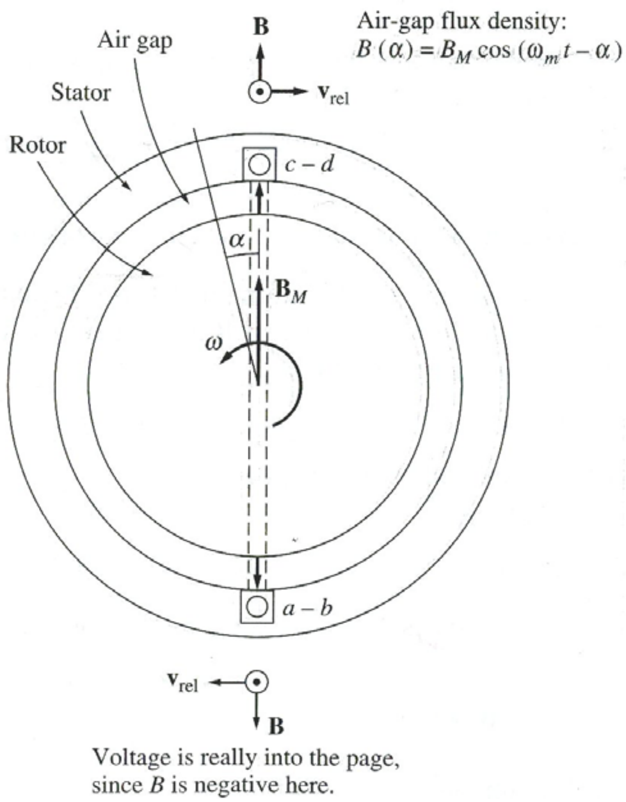
In practise, this winding **distribution** is **not possible** due to the **finite number of slots and integer number of conductors** are possible in each slot.

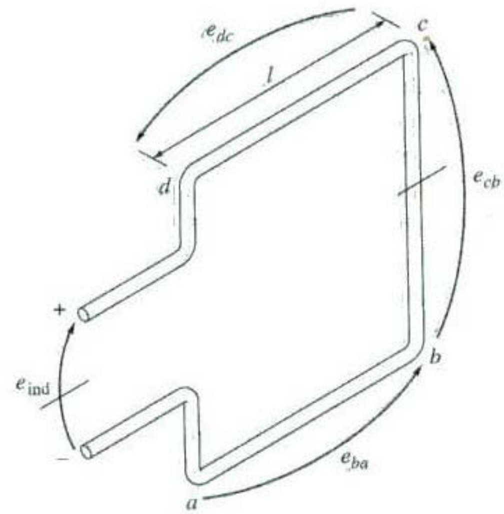
Therefore the actual flux distribution will consist of a fundamental sinusoidal component plus harmonics.

4.4 Induced voltage in AC machines

The rotating field can induce voltages in the three-phase windings.

The induced voltage in a coil on a two-pole stator





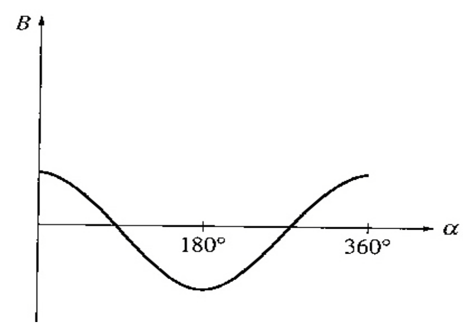


Figure above shows a **rotating rotor** with **a sinusoidally distributed magnetic field** in the **centre of** a **stationary coil**.

Assumptions:

- air gap flux density vector **magnitude**, B **varies sinusoidally** with mechanical angle
- **direction** of $\bar{B}$ always **radially outward**

The magnitude of the air gap flux density vector at a point **around the rotor** is given by:

$$B = B_{\mathrm{M}} \cos \alpha$$

where α = angle measured from the direction of peak rotor flux density.

Since the rotor is rotating within the stator at an angular velocity ω_m, then B_{gap} **around the stator** is:

$$B_{\text{gap}} = B_{\text{M}} \cos(\omega t - \alpha)$$

We know that the voltage induced in **a moving coil** inside a **stationary field** is

$$e_{\text{ind}} = (\bar{v} \times \bar{B}) \cdot \bar{l}$$

where:

$\bar{v}$ = velocity of the wire relative to the magnetic field
$\bar{B}$ = magnetic flux density vector
$\bar{l}$ = length of conductor in the magnetic field

However, we have a **stationary coil in a moving magnetic field**. Hence, to use the induced voltage equation above we must be in a frame of reference where magnetic field appears to be stationary, i.e.

If we "sit on the magnetic field", the field will appear stationary and the sides of the coil will appear to go by at an apparent velocity $\bar{v}_{\text{rel}}$, and the induced equation above can be applied.

The total voltage induced in the coil will be the sum of voltages induced in each of the four sides.

Segment *ab*	• $\alpha = 180°$ • $\bar{B}$ is directed radially outward • $(\vec{v} \times \vec{B})$ points ___________________ $e_{\text{ba}} = (\bar{v} \times \bar{B}) \cdot \bar{l}$ $= vBl$ $= -v[B_{\text{M}} \cos(\omega_{\text{m}} t - 180°)]l$ = _________________
Segment *bc*	• $\bar{v} \times \bar{B}$ is perpendicular to $\bar{l}$ • $e_{\text{cb}} = (\bar{v} \times \bar{B}) \cdot \bar{l} = 0$
Segment *cd*	• $\alpha = 0°$ • $\bar{B}$ directed radially outward

	• $\bar{v} \times \bar{B}$ points ________________ $e_{\text{dc}} = (\bar{v} \times \bar{B}) \cdot \bar{l}$ $= vBl$ $= v[B_{\text{M}} \cos(\omega_m t) l$ $=$__________________
Segment *da*	• $\bar{v} \times \bar{B}$ is perpendicular to $\bar{l}$ • $e_{\text{ad}} = (\bar{v} \times \bar{B}) \cdot \bar{l} = 0$

Hence, **total induced voltage on the coil**:

$$\begin{aligned} e_{\text{ind}} &= e_{\text{ba}} + e_{\text{dc}} \\ &= -vB_{\text{M}} l \cos(\omega_{\text{m}} t - 180°) + vB_{\text{M}} l \cos(\omega_{\text{m}} t) \\ &= 2vB_{\text{M}} l \cos \omega_{\text{m}} t \end{aligned}$$

Since$v = r\omega$, therefore

$$e_{\text{ind}} = 2(r\omega_{\text{m}})B_{\text{M}} l \cos \omega_{\text{m}} t$$

Finally, the flux passing through the coil is given by

$$\phi = 2rlB_{\text{M}}$$

Hence, the induced voltage can be expressed as

$$e_{\text{ind}} = \phi\omega \cos \omega t$$

Note: $\omega_{\text{m}} = \omega_e = \omega$ since it is a two-pole stator.

Or if the stator coil has N_C turns, then the total induced voltage of the coil is

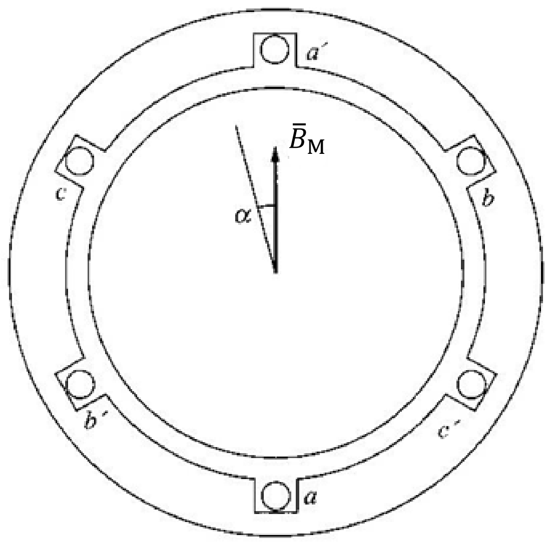
$$e_{\text{ind}} = N_{\text{C}}\phi\omega \cos \omega t$$

The induced voltage produced in the stator of this simple ac machine winding is sinusoidal with and amplitude which depends on the flux of the machine, the angular velocity ω of the rotor, and a constant depending on the construction of the machine (N_C is a simple case).

Note that: This derivation goes through the **induced voltage in the stator** when there is **a rotating magnetic field produced by the rotor**.

Induced voltage in a three-phase set of coils

If now we have **three coils** each having $\boldsymbol{N_C}$ **turns** placed around the rotor magnetic field,



Voltages induced in each coil will be same in magnitude but differ in phase by 120°.

The resulting voltages in each of the coil are:

$$e_{aa'} = N_C\phi\omega \sin \omega t$$
$$e_{bb'} = N_C\phi\omega \sin(\omega t - 120°)$$
$$e_{cc'} = N_C\phi\omega \sin(\omega t - 240°)$$

The equation illustrates that,

a <u>three-phase set of currents</u> flowing in the stator → generates a <u>uniform rotating magnetic</u> field in the stator.

and

a <u>uniform rotating magnetic field</u> produced by the stator → generates a <u>three-phase set of voltages</u> in the stator

Referring back to the induced voltage derived earlier…

Maximum induced voltage achieved when

sin (…) = 1

Hence, the **peak voltage in any phase** of the three-phase stator is:

$$E_{\text{max}} = N_{\text{C}}\phi\omega$$
or
$$E_{\text{max}} = 2\pi N_{\text{C}}\phi f$$

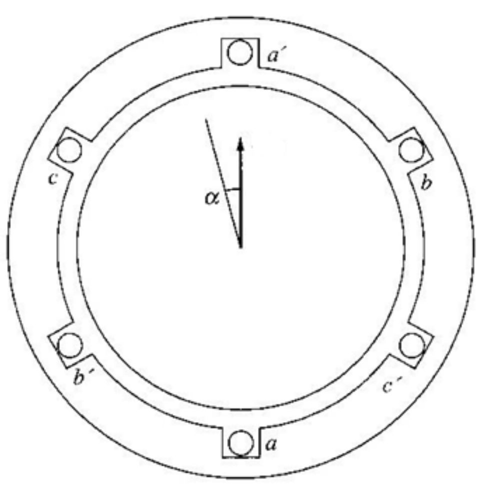
Finally, the **rms voltage** at **any phase** of the three-phase stator:

$$E_{\text{A}} = \frac{2\pi}{\sqrt{2}} N_{\text{C}}\phi f$$

Note: this is the **induced voltage at each phase**, as **for the line-line voltage values, it** will **depend upon** how the **stator windings** are **connected**, whether as **Y** **($\sqrt{3}E_{\text{A}}$) or Δ** ($\mathbf{E_A}$).

Example

The following information is known about a simple two-pole generator in Fig.



The peak flux density of the rotor magnetic field is 0.2 T, and the mechanical rate of rotation of the shaft is 3600 r/min. The stator diameter of the machine is 0.5 m, its coil length is 0.3 m, and there are 15 turns per coil. The machine is Y-connected.

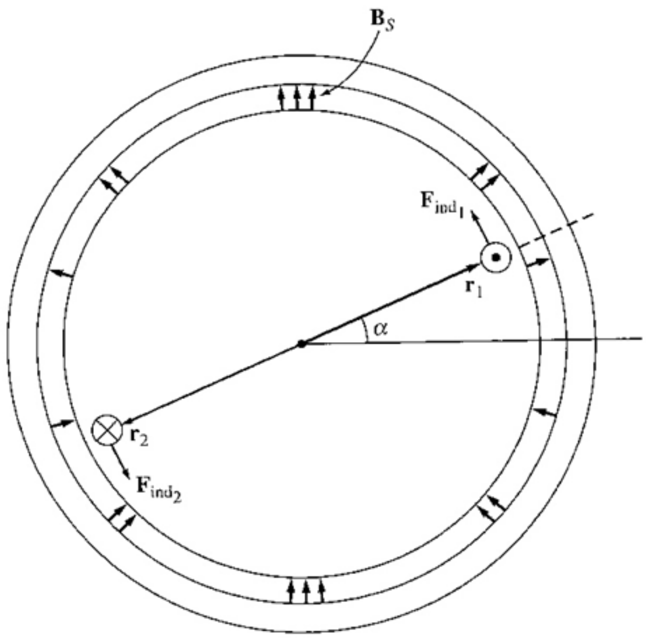
a) What are the 3-phase voltages of the generator as a function of time?
b) What is the rms phase voltage of this generator?
c) What is the rms terminal voltage of this generator?

4.5 Induced torque in an AC machine

In ac machines under normal operating conditions, there are two magnetic fields:

1) A magnetic field from the rotor circuit
2) A magnetic field from the stator circuit

Figure below shows a **simplified ac machine** with a **single coil of wire** mounted **on** the **rotor**.



$|\mathbf{B}_S(\alpha)| = B_S \sin\alpha$

The **stator flux density distribution**:

$$B_s(\alpha) = B_S \sin\alpha$$

$\boldsymbol{B_s(\alpha)}$ is positive when the flux density vector points radially outward from the rotor surface to the stator surface.

The torque produced in the rotor is obtained by analysing the force and torque on each of the two conductors separately:

	Induced force	Torque on conductor
Conductor 1	$\bar{F} = i(\bar{l} \times \bar{B})$ $= ilB_S \sin\alpha$ (direction shown in fig.)	$\tau_{ind,1} = (\bar{r} \times \bar{F})$ $= rilB_S \sin\alpha$ (counterclockwise)
Conductor 2	$\bar{F} = i(\bar{l} \times \bar{B})$ $= ilB_S \sin\alpha$ (direction shown in fig.)	$\tau_{ind,1} = (\bar{r} \times \bar{F})$ $= rilB_S \sin\alpha$ (counterclockwise)

Therefore, **the total torque on the loop**:

$$\tau_{ind} = 2rilB_S \sin\alpha$$

However, there are **two known facts**:

1. **Current** *i* flowing **in the rotor produces** a **magnetic field** on its own.

   Direction of field: is given by the right-hand rule
   Magnitude of **its magnetising intensity:** $H_r \propto i$

2. **Angle between** the **peak stator flux density** $\bar{B}_S$ and **peak rotor magnetising intensity** $\bar{H}_R$ is γ.

   Furthermore,

$$\gamma = 180° - \alpha$$
$$\sin\gamma = \sin(180° - \alpha) = \sin\alpha$$

By combining these two facts, **the torque on the loop** can be expressed as

$$\tau_{ind} = KH_R B_S \sin\alpha$$

or

$$\tau_{ind} = K(H_R \times B_S)$$

where *K* = constant dependent on machine construction.

Since $\boldsymbol{B}_{\mathbf{R}} = \boldsymbol{\mu}\boldsymbol{H}_{\mathbf{R}}$, the equation can be re-expressed as

$$\boldsymbol{\tau}_{\boldsymbol{ind}} = \boldsymbol{k}(\boldsymbol{B}_{\mathbf{R}} \times \boldsymbol{B}_{\mathbf{S}})$$

where $\boldsymbol{k} = \boldsymbol{K}/\boldsymbol{\mu}$.

The **net magnetic field** density will be the **vector summation** of the rotor and stator fields (assuming **no saturation**):

$$\bar{B}_{\mathrm{net}} = \bar{B}_{\mathrm{R}} + \bar{B}_{\mathrm{S}}$$

Hence, an equivalent expression for the induced torque is obtained:

$$\tau_{\mathrm{ind}} = k\bar{B}_{\mathrm{R}} \times (\bar{B}_{\mathrm{net}} - \bar{B}_{\mathrm{R}})$$

$$\tau_{\mathrm{ind}} = k(\bar{B}_{\mathrm{R}} \times \bar{B}_{\mathrm{net}}) - k(\bar{B}_{\mathrm{R}} \times \bar{B}_{\mathrm{R}})$$

$$\tau_{\mathrm{ind}} =$$

The magnitude of this expression is:

$$\tau_{\mathrm{ind}} = kB_{\mathrm{R}}B_{\mathrm{net}} \sin \delta$$

where δ is the angle between $\bar{B}_{\mathrm{R}}$ and $\bar{B}_{\mathrm{net}}$.

The value of *k* is unimportant in the **qualitative study** of torque in ac machines.

4.6 AC Machine Power Flow and Losses

The losses in ac machines can be divided into four basic catergoris:

a) Electrical or copper losses
 — Resistive heating losses that occur in stator and rotor windings of the machines
 — Stator copper loss $P_{\mathrm{SCL}} = 3I_{\mathrm{A}}^2R_{\mathrm{A}}$; Rotor copper loss $P_{\mathrm{RCL}} = I_{\mathrm{F}}^2R_{\mathrm{F}}$

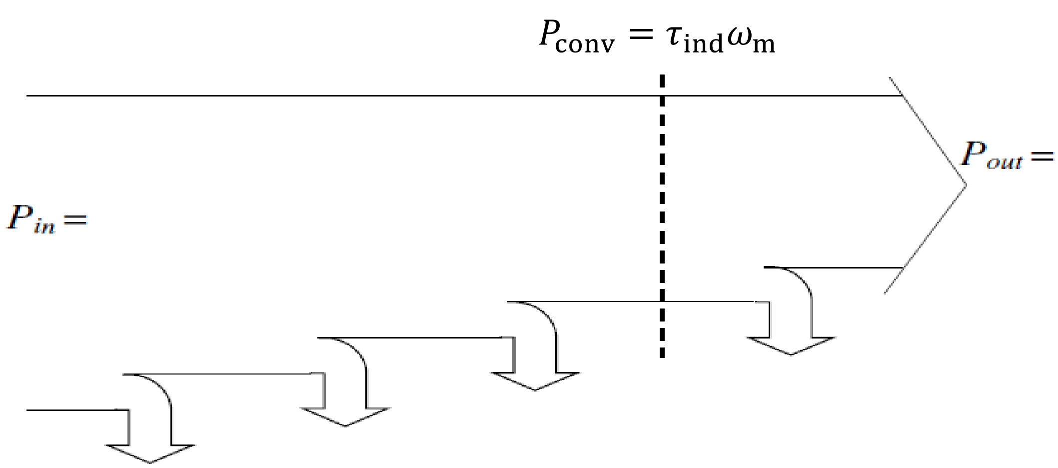
b) Core losses

 —Hysteresis and eddy current loss in the metal or the motor

c) Mechanical losses
 — Losses caused by the friction of the bearings
 — Losses caused by friction between the moving parts of the machines and the air inside the motor's casing
 — Known as the *no-load rotational loss*

d) Stray losses
 —Losses that cannot be accounted for

The Power-Flow Diagram



Power-flow diagram for ac generator

In the case of ac motors, the power flow diagram is simply reversed.